Relativistic and QED corrections to the metastable states of the antiprotonic helium atoms

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Antiprotonic helium atom ($\bar{p}\mathrm{He}^+$) is an exotic three-body system consisting of an antiproton ($\bar{p}$), a helium nucleus (He^{2+}) and an electron (e^-). The transition wavelengths were measured with precise laser spectroscopy (see [1], further references therein) for the metastable states. The data are very challenging to the few-body theory since it is necessary to accomplish very high precision in an energy-level calculation managing large interacting angular momenta of 30–40. Moreover the relativistic and QED effects exceed this accuracy [2]. To calculate the relativistic effects, we start with the Breit equation. Using Pauli approximation, we obtain the three-body Hamiltonian H for the Schödinger equation,

$$H = \sum_i \frac{\mathbf{p}_i^2}{2m_i} + \sum_{i>j} \frac{z_i z_j}{r_{ij}} - K_{\mathrm{cm}} + \alpha^2 H_{\mathrm{rel}} + \alpha^3 H_{\mathrm{QED}}, \tag{1}$$

with

$$\begin{aligned} H_{\mathrm{rel}} = -\sum_i \frac{\mathbf{p}_i^4}{8m_i^3} + \sum_{i>j} \Bigg[& -\frac{z_i z_j}{2}\left(\frac{1}{m_i^2}+\frac{1}{m_j^2}\right)\pi\delta(\mathbf{r}_{ij}) - \frac{z_i z_j}{2m_i m_j}\left(\frac{\mathbf{p}_i\cdot\mathbf{p}_i}{r_{ij}} + \frac{(\mathbf{r}_{ij}\cdot\mathbf{p}_i)(\mathbf{r}_{ij}\cdot\mathbf{p}_j)}{r_{ij}}\right) \\ & + \frac{(z_i - 2\mu_i)z_j}{2m_i^2}\frac{(\mathbf{r}_{ij}\times\mathbf{p}_i)\cdot\mathbf{s}_i}{r_{ij}^3} - \frac{z_i z_j}{m_i m_j}\frac{(\mathbf{r}_{ij}\times\mathbf{p}_i)\cdot\mathbf{s}_j}{r_{ij}^3} \\ & - \frac{8\pi}{3}\frac{\mu_i\mu_j}{m_i m_j}(\mathbf{s}_i\cdot\mathbf{s}_j)\delta(\mathbf{r}_{ij}) + \frac{\mu_i\mu_j}{m_i m_j}\left((\mathbf{s}_i\cdot\mathbf{s}_j) - 3\frac{(\mathbf{r}_{ij}\times\mathbf{s}_i)(\mathbf{r}_{ij}\times\mathbf{s}_j)}{r_{ij}^5}\right)\Bigg], \end{aligned} \tag{2}$$

$$H_{\mathrm{QED}} = -\frac{4}{3}\left(\frac{19}{30} - 2\ln\alpha + \ln K_0\right)\Big(z_\alpha z_e \delta(\mathbf{r}_{\alpha e}) + z_{\bar{p}} z_e \delta(\mathbf{r}_{\bar{p}e})\Big), \tag{3}$$

where $\mathbf{p}_i$, m_i z_i, μ_i and $\mathbf{r}_{ij}$ are the momentum of ith particle, its mass, its electric charge, its magnetic dipole moment and the relative vector between ith particle and jth particle. The value $\ln K_0 = 4.4$ is the Bethe logarithm. We subtract the kinetic operator K_{cm} of the center of mass system from the Hamiltonian. The operator H_{QED} corresponding to the one-loop vacuum polarization and self-energy is added to the Hamiltonian. The total wave function is given by

$$\Phi_{vJFM} = \sum_K C_K \left[\left[\Psi_{vJ}^{\bar{p}\mathrm{He}^+} \otimes \xi_e\right]_K \otimes \xi_{\bar{p}}\right]_{FM}, \tag{4}$$

where $\Psi_{vJ}^{\bar{p}\mathrm{He}^+}$ is a non-relativistic three-body wave function, C_K is an expansion coefficient and ξ is a spin function. The non-relativistic wave functions and energies are calculated with a non-adiabatic coupled rearrangement channel method [3] diagonalizing the non-relativistic part of the Hamiltonian. The three rearrangement channels are shown in Ref. [4]. The relativistic and QED corrections are calculated with the first order perturbation theory. In Table 1, the transition wavelengths and uncertainty of the antiproton mass (charge) are listed. The relativistic and QED collection reduced the discrepancy between the calculated and observed values by about 40 ppm. To estimate the uncertainty of $\bar{p}$ mass from the experimental data, we recalculated the wavelength using the $\bar{p}$ mass and charge scaled with $1 + \Delta x$ against the p mass and charge, $m_{\bar{p}} = (1 + \Delta x)m_p$ and $e_{\bar{p}} = (1 + \Delta x)e_p$. The value Δx was estimated using an uncertainty of the experimental wavelengths $\Delta\lambda_{\mathrm{expt}}$,

$$\Delta x = \frac{\Delta m_{\bar{p}}}{m_p} = \Delta\lambda_{\mathrm{expt}} \left(\frac{d\lambda}{dx}\right)^{-1}_{\mathrm{cal}}. \tag{5}$$

We reduced uncertainty of the $\bar{p}$ mass (or equivalently charge) by two orders of magnitude [5].

We also calculated the fine and hyperfine splitting of the metastable states [7] for the forthcoming experiment using microwave resonance at CERN. The values agree well with other calculation [6]. We obtained the transition wavelengths 713.5819 nm for (J=35, v=1, K=J+1/2) $\rightarrow$ (34, 3, J+1/2) and 713.5812 nm for (J=35, v=1, K=J–1/2) $\rightarrow$ (34, 3, J–1/2), and obtained the transition frequencies 12.899 GHz for (J=35, v=1, K=J–1/2, F=J) $\rightarrow$ (35, 1, J+1/2, J+1) and 12.927 GHz for (J=35, v=1, K=J–1/2, F=J–1) $\rightarrow$ (35, 1, J+1/2, J).

Table 1: Transition wavelengths, λ_{NR}: pure Coulomb calculation, λ_{C}: calculation with relativistic and QED corrections and λ_{E}: experimental values. Δx is the uncertainty of $\bar{p}$ mass.

(J_i,v_i)–(J_f,v_f)	λ_{NR} (nm)	λ_{C} (nm)	λ_{E} [1] (nm)	Δx
(35,3)–(34,3)	597.2290	597.2573	597.2570 ± 0.0003	1×10^{-7}
(34,2)–(33,2)	470.7048	470.7220	470.7220 ± 0.0006	3×10^{-7}

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